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AURUM MÜHENDİSLİK SİSTEMLERİ VE MİMARLIK DERGİSİ
AURUM JOURNAL OF ENGINEERING SYSTEMS AND ARCHITECTURE

Cilt 5, Sayı 2 | Kış 2021
Volume 5, No 2 | Winter 2021, 261-278

ARAŞTIRMA MAKALESİ / RESEARCH ARTICLE

RELIABILITY OF DESIGN OF FLOOR SLAB SYSTEMS USING PLASTIC MOMENT DISTRIBUTION METHOD

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GELİŞ TARİHİ/RECEIVED DATE: 08.06.2021 KABUL TARİHİ/ACCEPTED DATE: 04.12.2021

Abstract
This research study the reliability of design continuous floor slabs with varying end conditions under effected ultimate Loads by plastic theory. This paper has adopted approach using plastic moment distribution method in the design of solid slab. 16 rectangular slabs have same depth but different end conditions and dimension in x and y- directions were investigated. The results obtained from the continuous solid slabs by plastic moment distribution method achieved the conditions of equilibrium, mechanisms, and plastic moment and its was compared with computer program adopted elastic theory method under loading combination ultimate limit state and serviceability limit state. The results are compared according to the parameters of the British Code (BS 8810) allow that any value of bending moments by plastic method must be not less than 30 % of elastic moment at supports and at span (maximum sagging moments) in plastic are high in elastic analysis. After comparing, all the obtained results showed that they are in agreement with the determinants of the British code. The method adopted in this study gave the values of the plastic moment (MP), at which the plastic hinge (PH) is formed and true mechanism of failure.

Keywords: Reliability, Moment Plastic, Solid slab, Plastic hinge, Mechanism

PLASTİK MOMENT DAĞILIM YÖNTEMİ KULLANILARAK TASARIM ZEMİN DÖŞEMESİNİN GÜVENİLİRLİĞİ

Özet

Bu araştırma, plastik teori ile etkilenen nihai yükler altında değişen son koşullara sahip tasarım sürekli döşeme plakalarının güvenilirliğini incelemektedir. Bu makale, katı döşemenin tasarımında plastik moment dağılımı yöntemini kullanan bir yaklaşım benimsemiştir. Aynı derinliğe sahip fakat x ve y-doğrultularında farklı boyut ve son koşullarda olan 16 dikdörtgen döşeme araştırılmıştır. Plastik moment dağılım yöntemi ile sürekli katı levhalardan elde edilen sonuçlar, denge, mekanizma ve plastik moment koşullarını sağladı ve bu, bilgisayar programı tarafından benimsenen elastik teori yöntemi ile yükleme kombinasyonu nihai sınır durumu ve hizmet verebilirlik sınır durumu altında karşılaştırıldı. Sonuçlar, İngiliz Kodunun (BS 8810) parametrelerine göre karşılaştırıldığında, plastik yöntemle herhangi bir eğilme momenti değerinin desteklerdeki elastik momentin %30'undan az olmaması gerektiğine ve elastik analizde plastikte açıklıkta (maksimum sarkma momentleri) yüksek olmasına izin verir. Karşılaştırıldıktan sonra, elde edilen tüm sonuçlar İngiliz kodunun belirleyicileri ile uyumlu olduklarını göstermiştir. Bu çalışmada benimsenen yöntem, plastik mafsalın (PH) oluştuğu ve göçme mekanizması oluştuğunda plastik moment (MP) değerlerini vermiştir.

Anahtar Kelimeler: Güvenilirlik, Plastik Moment, Solid Döşeme, Plastik mafsal, Mekanizma

1. INTRODUCTION

There are several possible approaches to analysis and design of reinforcing concrete systems of floor slab. Many of these approaches are depending on the elastic theory, this approach is known that, for its confidence on the assumption of the stresses occur in the structure due to loads fall with in elastic limits of the material using, So the deflection will be insignificant. In order to know what happens when a collapse and how behavior of the structures when stresses in the material when exceeding the limits of elasticity. It can be seen that this philosophy is embodied in plastic method of analysis and design (Moy, 2013). Slabs are considered one of the most important structural and basic elements in buildings. And it represents a large proportion of the areas of those buildings They are constructed mostly of reinforced concrete or other materials like composite steel with concrete or plate steel and that are considered to have high weights (Vančik and Jirásek, 2016). Nowadays, more than 65% of the structures in the world are made of concrete (Alasam, 2006).

It was mentioned by (Chen and Sohal, 1995) plastic structure design has various advantages over elastic design, the most significant of which are simplified methods, cost savings, and a more realistic simulation of structure behavior.

It is explained by (Farouk et al., 2018) that the plastic theory is a simple method for analyzing an element while it is in the plastic stage. This method can be used to predict both the failure load and the failure mechanism, as well as to evaluate the elastic redistribution moments in indeterminate structures. Plastic analysis is based on the idealization of the stress–strain curve as elastic-perfectly plastic in plastic theory.in this studies the suggested structural model to present a simple solution for steel beams in plastic stage. the present model was verified using comparing results with experimental results which made by other and the comparing showed matching in results.

(Horne and Fruchtlander, 1954) They pointed out three basic requirements for the distribution of the bending moment, which represents a state of collapse, which are:

i. Equilibrium condition (The distribution of bending moment shall be balanced with the loads applied
ii. condition of collapse mechanism (There need to be enough plastic hinge to create a mechanism for the whole structure or part of it.
iii. plastic moment condition (There is no point in exceeding the plastic moment)

The researchers adopted plastic moment distribution method to design of continuous beam and discussed the results with that obtained by elastic distributing method. Comparisons between elastic and plastic moment distribution processes according to carry over moment at center of span and left, right hand as shown as in Table 1.

Table 1. Carry Over Moment for Elastic and Plastic Distribution Moment

Method	Left-hand	Central	Right-hand
Elastic	1→	(1/4)→	1/2
	1/2	←(-1/4)	←1
Plastic	←1	(1/2)→	0
	0	←(1/2)	←1

1.1 Rigid Plastic and Plastic Moment

As shown by (Moy, 2013), when the stresses reach the stage of yielding in the upper and lower fibers of the section and during an increase in the external loads, it is matched by an increase in the strain while the same stress value remains, and then the entire section becomes in the yield stage, then the plastic is formed as an approach and the moment at this stage is the plastic moment (MP) and the form of failure is formed, which is called mechanism. Figure.1 show the plastic hinge and plastic moment.

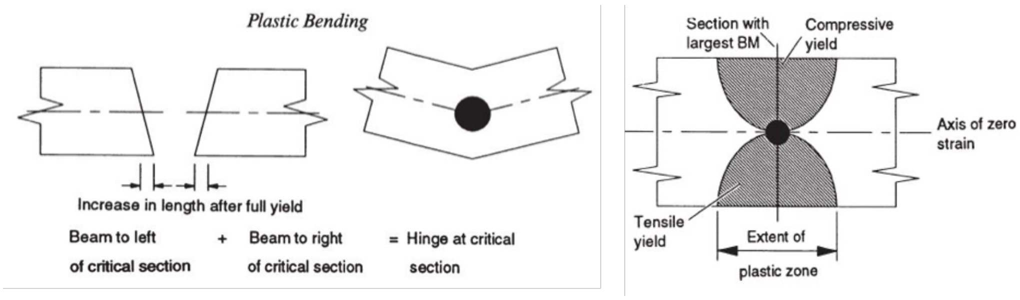


Figure 1. Show the plactic hinge and plastic moment (Stuart S. J. May - 1996)

1.2 Problem Statement

Slabs are considered one of the most important structural and basic elements in buildings. And it represents a large proportion of the areas of those buildings they are constructed mostly of reinforced concrete or other materials like composite steel with concrete or plate steel and that are considered to have high weights. Nowadays, more than 65% of the structures in the world are made of concrete (Alasam, 2006). Therefore, it is very important to use a design method that ensures weight reduction, and at the same time it is safe, it is also known to us that the most method of analysis and design is based on the elastic properties of materials. The elastic design does not take into account the strength of the material beyond the elastic stress (Singh, 2011). Therefore, the structure designed according to this method will be heavier than that designed by plastic methods.

1.3 Objective of This Study

The aim of this study is to know the structural behavior of the solid slab after the elasticity stage and when it is exposed to the ultimate loads and reaches the plasticity stage and to design the slab under the influence of the plastic moment, And to find a simple approach to calculate those moments and compare the results in a known, reliable and approved mothed from the code and find out their reliability and validity through obtaining the result through the Adopting a two-way multi-spans slabs with a constant thickness exposed to a uniform distribution load.

1.4 Scope of the Study

This study taken in to considerations the parameters influential of the behaviors of plastic moment distribution for solid slabs:

i. Continuous Solid slabs with different end condition for both direction one way and tow way slab.
ii. Applied uniform distribution load (Dead load & Live load)
iii. The thickness of all spans is constant for all different end condition
iv. One meter unit long middle strip in x- direction and y – direction

1.5 Methodology

a. Developing hand approach model to calculation flexural moment at ultimate load (collapse load) by using plastic moment distribution method for reinforced concrete element subjected the uniform distribution load and make design.
b. Using structural analysis programs (Robot 2020) to analysis and design the model to validate hand approach model.
c. Collection the data, and compare results, and find out the reliability of the model proposed by the researcher.

2. Plastic Design Using Moment Distribution Method

There are two approaches in designing structures:

i. Analysis Approach: in this approach geometry, loads and cross sections are data, stresses and deformation are objectives to be satisfied

ii. Synthesis Approach: Geometry of structure, external loads and materials properties (allowable stress, yield stresses, modulus of elasticity) are known.

Aim is directly obtaining the dimensions of section and amount of area steel for the member, the limitations stresses and displacement criteria given by the valid codes of practice are not exceeded. In any structure the internal forces "member forces "develop under a set of external loads and must satisfy the following equations (Hibbeler, 2012).

a. Equilibrium Equations
b. Constitutive Equations (stress – strain relations)
c. Continuity Equations (or conditions)

Any state of internal forces of structure in equilibrium under a given loading satisfy the equation in (a and b) above and the continuity conditions are not satisfy , plasticity develops at certain places and plastic hinges occur. Theses hinges rotate to establish continuity conditions. During this stage the internal forces change to keeping on the equilibrium equation this event is called "redistribution of internal forces "the main operation in "synthesis approach "is to adjust the maximum bending moments to satisfy the equilibrium equations at the desired failure mechanism, the structure is reanalyzed under the service loads and displacements and early developing of plastic hinges are checked. synthesis approach starts with fixed end moment as the first step:

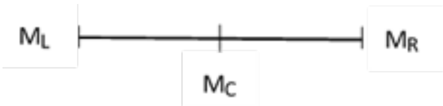


Figure 2. Fixed End Moments Locations

M_L : Left end fixed moment
M_R : Right end fixed moment
M_C : Bending moment in the middle
These moments are in equilibrium with external loads. In order to satisfy joint equilibrium equations
$M_L \rightarrow M_L + \delta M_L$
$M_R \rightarrow M_R + \delta M_R$
$M_C \rightarrow M_C + \delta M_C$

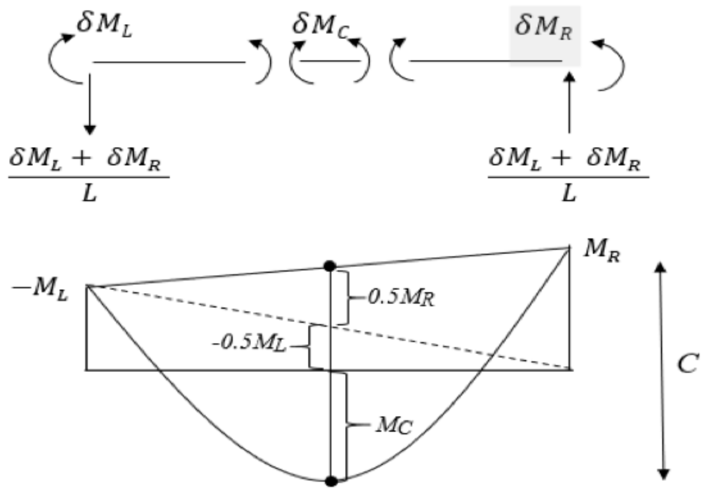


Figure 3. Free Bending Moment Diagram

C : *free bending moment*
Before change: $-0.5\,M_L + M_C + 0.5\,M_R = C$
After change:
$-0.5(M_L + \delta M_L) + (M_C + \delta MC) + 0.5(MR + \delta MR) = C$
$\underbrace{-0.5M_L + M_C + 0.5M_R}_{C} - 0.5M_L + \delta M_C + 0.5M_R = C$

Differential Equilibrium Equation:

$$M_L + 2\delta M_C + M_R \ldots (1)$$

After balancing of left and right joint δM_L, δM_C and δM_R must satisfy the above Differential Equilibrium Equation . A distribution coefficient of bending moments given below automatically satisfy this equation.

Table 2. Show distribution plastic carry over of bending moments

(Left – hand) δM_L	Central - δM_C	(Right – hand) δM_R
1	$+\frac{1}{2}$	0
0	$-\frac{1}{2}$	1
1	0	1
$M_L + \delta_{ML}$	$M_C + \delta M_C$	$M_R + \delta M_R$

Example 1: Design the continuous slab as a beam with width 1 m below under The given loading (*w*) KN/m by plastic distribution method.

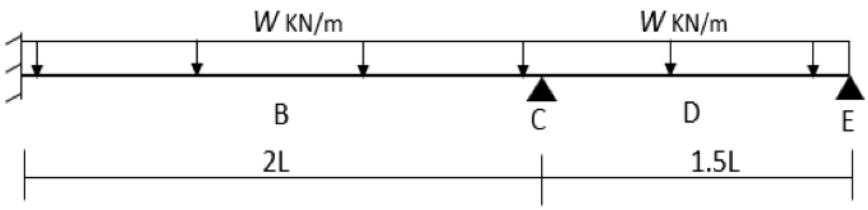


Figure 4. Dimensions and Loads

Solution: Calculation the steps below

1. Fixed End Moment

Span AC $FEM_{A-C} = -\frac{w\,(2l)^2}{12} = -\frac{w\,l^2}{3}$, $FEM_{C-A} = +\frac{w\,l^2}{3}$

Span CE $FEM_{C-E} = -\frac{w(1.5l)^2}{12} = -\frac{3w\,l^2}{16}$, $FEM_{E-C} = +\frac{3w\,l^2}{16}$

2. Free bending moment (C)

Span AC $C = \frac{w(2l)^2}{8} = \frac{w\,l^2}{2}$

Span CE $C = \frac{w(1.5l)^2}{8} = \frac{9w\,l^2}{32}$

3. Distribution moment by plastic method with carry over moment below

When making distribution, the following conditions must be taken:

Step 1. For economy $M_A = M_B = M_{C\text{-Left}}$

Step 2. For joint equilibrium $M_{C\text{-Left}} = M_{C\text{-Right}}$

Step 3. At point E = simple support $M_E = 0$

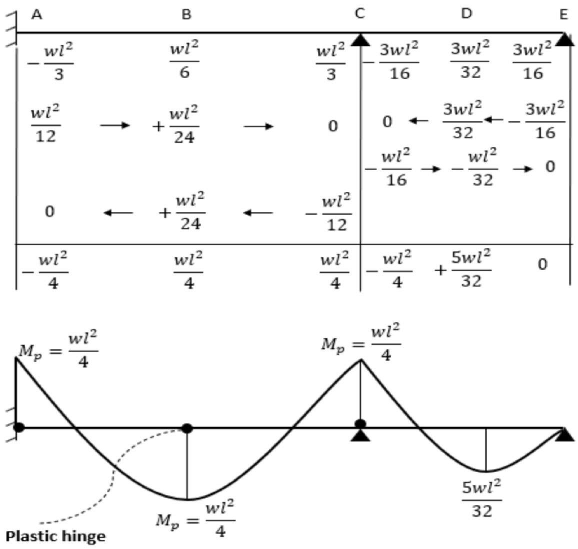


Figure 5. Moment Distribution and Location of Plastic Hinge

Before distribution the moment in mid span

For span AC:

M_B in mid-span = free bending moment (C) – FEM (Left +Right)/2

$$M = \frac{w\,l^2}{2} - (\frac{w\,l^2}{3} + \frac{w\,l^2}{3})/2 = \frac{wl^2}{6}$$

For span CE:

$$M = \frac{9w\,l^2}{32} - (\frac{3w\,l^2}{16} + \frac{3w\,l^2}{16})/2 = \frac{3wl^2}{32}$$

For distribution moment we start with span has max free bending moment in this example take

Span AC $C = \frac{w(2l)^2}{8} = \frac{w\,l^2}{2}$

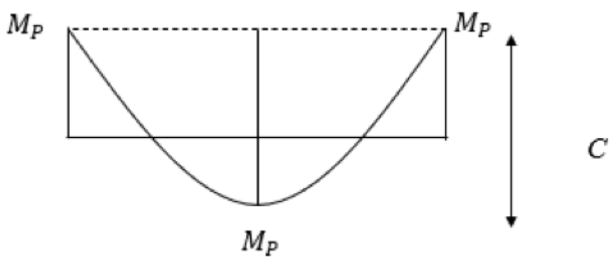


Figure 6. Span AC

C=2M_P

$$\frac{w\,L^2}{2} = 2M_P \rightarrow M_P = \frac{w\,L^2}{4}$$

We can see the final moment at point (A, B, C) are plastic moments and plastic hinge deformed and for point (D) didn't have plastic moment and value less than plastic moment finally the moment must be equal zero because real hinge .

2.1 Case Study

In this case study, we take the irregular dimension in y and x-direction and make a design of this case study by plastic moment distribution method. A numerical example will be taken 16 solid continuous slabs with different dimension as shown below:

Example 2

Design the floor slab shown in the figure below, carrying a live load 3 KN/m2 And wight of finishes 2 KN/m2 with compressive strength 30 MPa and steel yield strength fy 350 MPa.

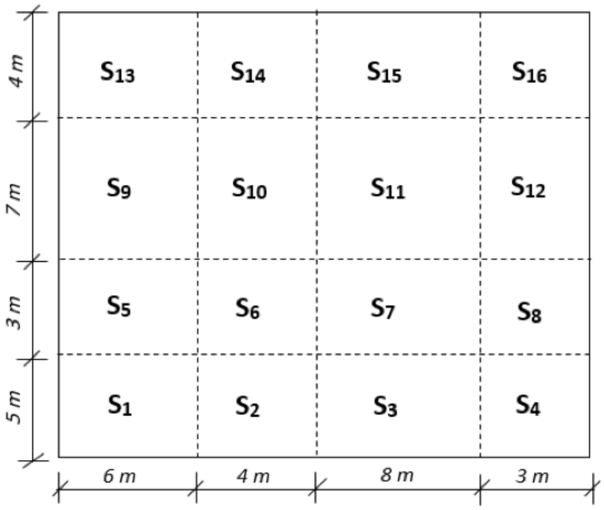


Figure 7. Layout of The Floor System

Solution:

Step 1 Calculation the thickness of slab

The biggest slab is S_{11} (7x8) m

$$h = 2 \times \frac{8000+7000}{180} = 166{,}66\ mm$$ take h=170 mm according to (ACI-318, 2014)

Step 2 Calculation the Load on the slab

Self-weight of slab: $0.170 \times 24 = 4.1$ KN / m^2

Weight of finishes (given): 2 KN / m^2

Total Dead load: 6.1 KN / m^2

Live load: 3 KN / m^2

$Wu = 1.2 \times D \times L + 1.4 \times L \times L$

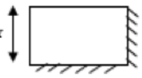
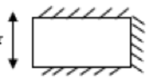
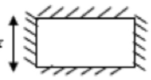
Total factored load, $Wu = (1.2 \times 6.1 + 1.6 \times 3) = 12.12\ KN / m^2$

For strip as beam 1m width load acting is 12.12×1 m=12.12 KN/m

Now we estimation the transferred load (w_x and w_y) of total load Wu = 12.12 KN/m in x and y direction by taking strip in mid of each span for all panels in both direction

Step 3 Calculation load transferred in x- direction and bending moments according to Table 3

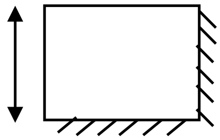
Table 3. Show Proportion of load in Each Direction Based on Grashof – Rankine

Supports Condition — *Simply supported*; ⁄⁄⁄⁄ *Continuous*	*Proportion of Load in each Direction*	
	w_x/w	w_y/w
 L_x	$\frac{r^4}{1+r^4}$	$\frac{1}{1+r^4}$
 L_x	$\frac{2r^4}{1+2r^4}$	$\frac{1}{1+2r^4}$
 L_x	$\frac{r^4}{1+r^4}$	$\frac{1}{1+r^4}$

Note r = Ly/Lx

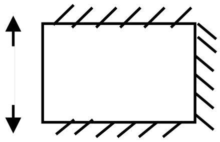
1.Strip (1) for slab panels (S1 , S2 , S3 , S4)

For panels (S1) and (S4): -



$$\frac{w_y}{W} = \left(\frac{1}{1+r^4}\right) \quad ...(2)$$

For panels (S2) and (S3): -



$$\frac{w_y}{W} = \left(\frac{2r^4}{1+2r^4}\right) \quad ...(3)$$

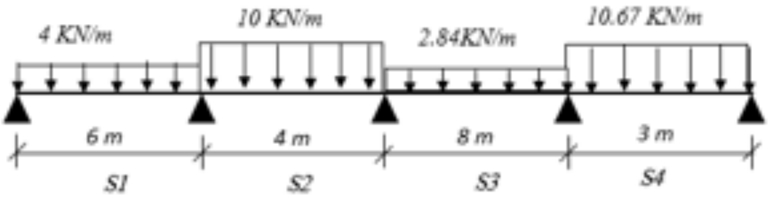


Figure 8. Floor Distributed Loads

Table 4. Bending Moment Plastic for Strip (1)

panels	Point	Lx	Ly	r	Share Load	wx KN/m	Span m	Free bending moment (C)	Fixed End Moment (KN.m)	Final moment by plastic distribution
S1	a	5	6	1.Şub	0.325	4	6	18	-12	0
	b								6	12
	c								12	**12**
S2	c	4	5	Oca.25	0.83	10	4	20	-13.33	-12
	d								Haz.67	Ağu.32
	e								13.33	Kas.36
S3	e	8	5	0.625	0.234	Şub.84	8	22.72	-15.15	-11.36
	f								Tem.56	Kas.36
	g								15.15	Kas.36
S4	g	3	5	Oca.67	0.88	Eki.67	3	12	-8	-11.36
	h								4	Tem.33
	i								8	0

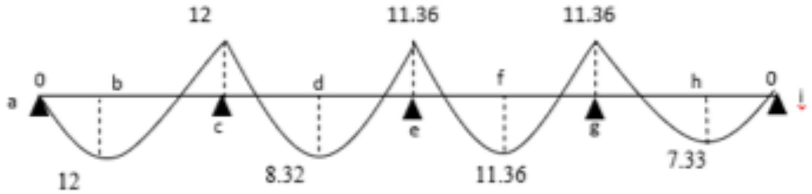


Figure 9. Moment Diagram

Figure 9. show the Full plastic moment MP = 11.36 KN.m All bending moment less than the plastic moment and the value 12 KN.m approximately equal plastic moment, so the mechanism of failure formed when the Plastic hinge occurs at points (b, c, e, f, g). for the other strips following the same procedure then we get the all results for 16 slabs panel as shown in figure below:

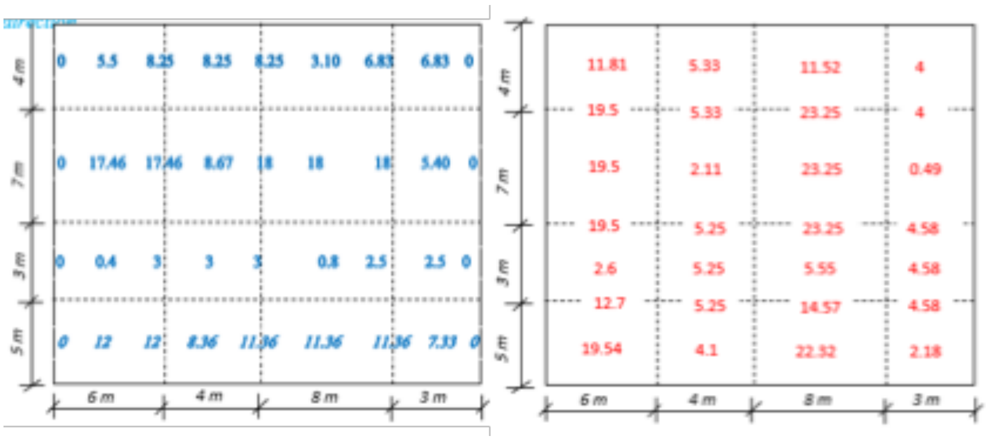


Figure 10. Distribution of Moments in the slabs for x and y – directions

2.2 The Model Slab in Robot Programs

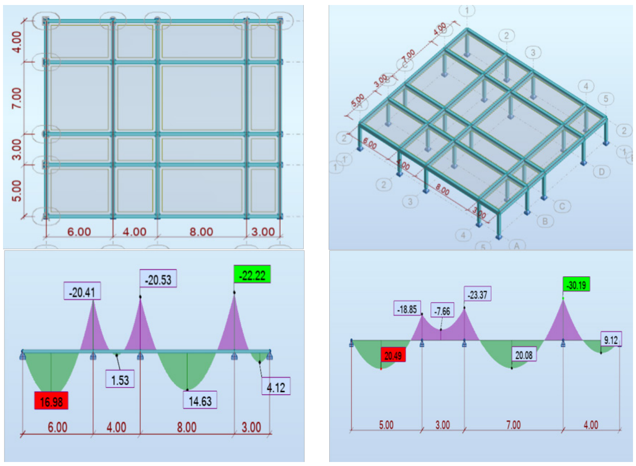


Figure 11. Distribution of Moments for strip in x and y-direction by Robot

3. RESULTS AND DISCUSSION

Plastic Moment distribution is performed for 16 solid continuous slabs with different dimension and boundary end conditions and varication all results with three requirements for plastic theory after that comparison the results with the same slabs using elastics moment distributing by computer programs (Autodesk Robot Structural Analysis Professional 2020). In this study The results obtained from the plastic analysis with the results of the program in elastic distribution were tested for the criteria and determinants of the British Code(British Standards Institution, 1997), and through comparison it is possible and check bending moment diagram to find reliability. based on to following:

1. At the joints beams and columns (supports)
2. In the maximum sagging moments
3. In the middle of the span (free bending moment)

3.1 At the Joint and Maximum Sagging Moment in x-direction

Table 5. Banding Moment strip 1

Slab	point	distance m	Bending Moment (KN.m/m)			
			SLS	ULS	Plastic	U.L.S Support moment 30 % reduce
S1	a	0	0	0	0	0
	b	3	-8.04	-10.91	-12	-10.91
	c	6	Kas.88	15.94	12	Kas.16
S2	d	9	-3.78	-4.89	-8.32	-4.89
	e	12	Eki.77	14.29	Kas.36	10.00
S3	f	15	-6.19	-8.27	-11.36	-8.27
	g	18	Eki.93	14.Haz	Kas.36	Eki.22
S4	h	21	-4.36	-5.81	-7.33	-5.81
	i	24	0	0	0	0

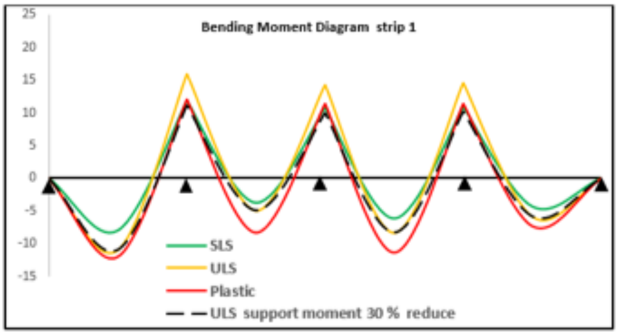


Figure 12. Moment Bending Diagram Strip 1

Table 6. Bending Moment strip 3

Slab	point	distance m	Bending Moment (KN.m/m)			
			S.L.S	U.L.S	Plastic	U.L.S support moment 30 % reduce
S5	a	0	0	0	0	0
	b	3	-12.87	-16.98	-17.46	-16.98
	c	6	15.35	20.41	17.46	14.29
S6	d	9	-1	-1.53	-4.27	-1.53
	e	12	15.44	20.53	18	14.37
S7	f	15	-11.08	-14.63	-18	-14.63
	g	18	16.81	22.22	18	15.55
S8	h	21	-3.1	-4.12	-5.4	-4.12
	i	24	0	0	0	0

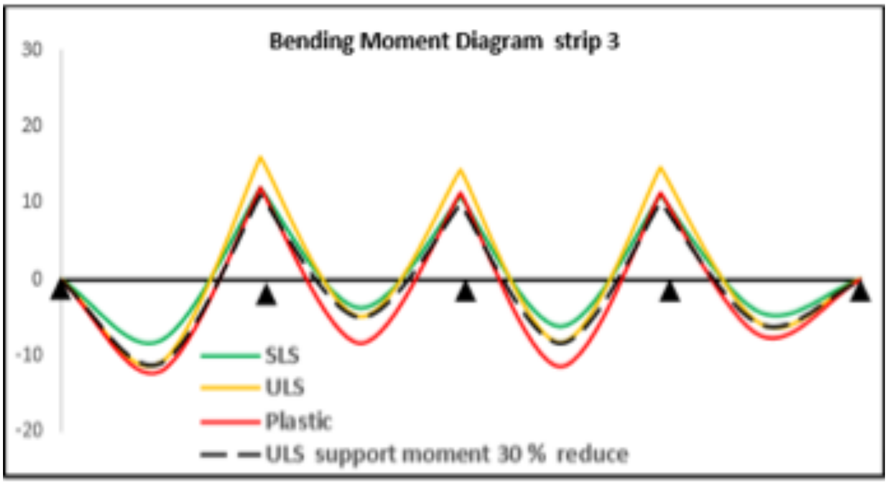


Figure 13. Moment Bending Diagram Strip 3

3.2 At the Joint and Maximum Sagging Moment in y -direction

Table 7. Bending Moment strip 5

slab	point	distance m	Bending Moment (KN.m/m)			
			S.L.S	U.L.S	Plastic	U.L.S support moment 30 % reduce
S1	a	0	0	0	0	0
	b	3	-13.15	-17.51	-19.54	-17.51
	c	6	Ara.91	17.19	12.Tem	12.Mar
S5	d	9	3.Ağu	5.May	2.Haz	5.May
	e	12	14.55	19.37	19.May	13.56
S9	f	15	-12.03	-16.05	-19.5	-16.05
	g	18	19.36	25.81	19.May	18.Tem
S13	h	21	-7.13	-9.54	-11.81	-9.54
	i	24	0	0	0	0

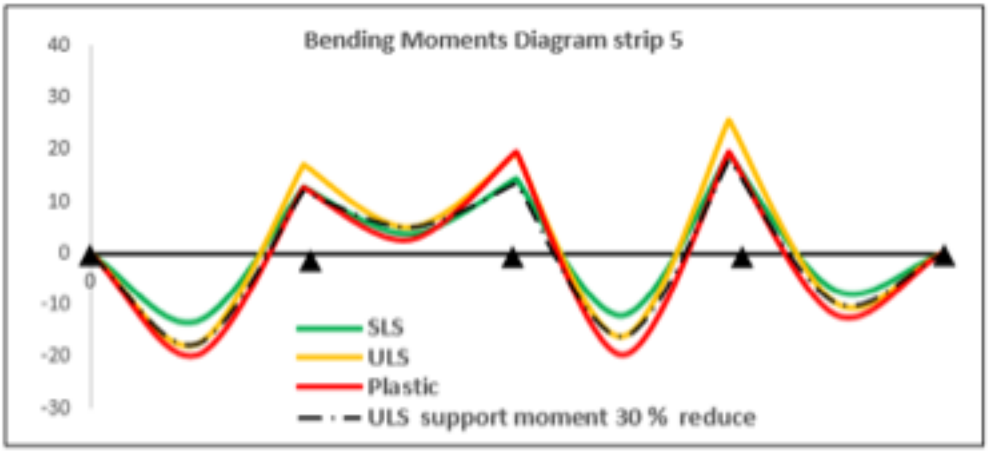


Figure 14. Moment Bending Diagram Strip 5

Table 8. Bending Moment strip 7

Slab	point	distance m	Bending Moment (KN.m/m)			
			S.L.S	U.L.S	Plastic	U.L.S support moment 30 % reduce
S3	a	0	0	0	0	0
	b	3	-15.37	-20.49	-23	-20.49
	c	6	14.14	18.85	14.57	13.20
S7	d	9	May.74	Tem.66	May.55	Tem.66
	e	12	17.56	23.Nis	23.25	16.38
S11	f	15	-15.08	-20.08	-23.25	-20.08
	g	18	22.57	30.19	23.25	21.13
S15	h	21	-4.73	-9.12	-11.52	-9.12
	i	24	0	0	0	0

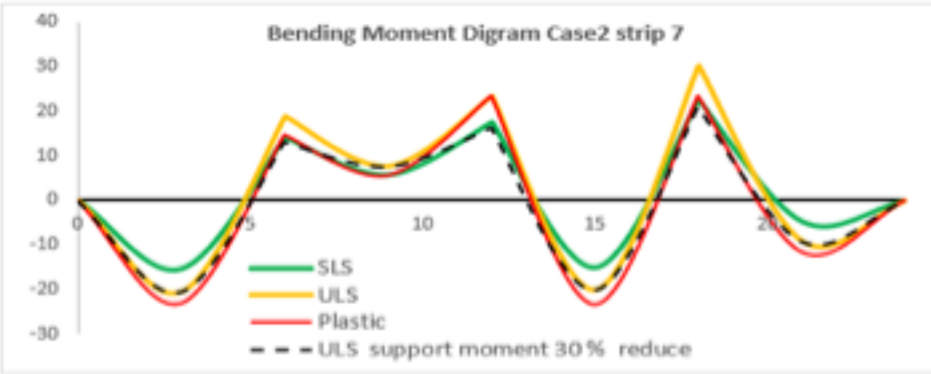


Figure 15. Moment Bending Diagram Strip 7 Case 2

4. CONCLUSION

In tables (5,6,7 and 8) and figures (12,13,14 and 15) as shown above represented the following:

1. Plastic: redistribution of B.M.D of loading at ultimate limit state in plastic analysis using suggesting model (plastic moment distribution method) in this study
2. SLS, ULS , ULS with 30% reduce : redistribution of B.M.D of loading at Serviceability and Ultimate limit state in elastic analysis by computer program (Robot 2020).
3. The presented model and the computer program was verified by comparing with British standard BS 8110 : Part 1 limits as shown by (Stark & Brekelmans, 1990) graphically in figure 16.

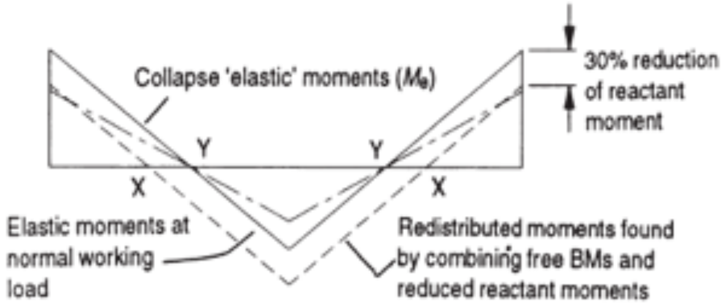


Figure 16. Show the B.M.D at Plastic, SLS, ULS, ULS 30 % reducer at supports

4. All results in presented approach in this study matching with code limits at supports the method gives results within the redistribution up to 30 % thus is usable in practice, at span sagging mo ment the method show the value of B.M.D is grater then ULS, so the redistribution by plastic method gives the true results.

5. REFERENCES

ACI-318. 2014. Building Code Requirements for Structural Concrete (ACI 318S-14) and Commentary (ACI 318SR-14). In American Concrete Institute.

Alasam, M. A. A. 2006. Yield Line Method Applied to Slabs with Different Supports. December.

British Standards Institution. 1997. Bs 8110-1:1997. British Standard Institution London, 1, 168.

Chen, W. F., & Sohal, I. 1995. Plastic Design and Second-Order Analysis of Steel Frames. In Plastic Design and Second-Order Analysis of Steel Frames. https://doi.org/10.1007/978-1-4613-8428-1

Farouk, M. A., Alzara, M., and Samir El-kady, M. 2018. Computing redistribution moments in the plastic stage by using linear analysis. Innovative Infrastructure Solutions, 3(1). https://doi.org/10.1007/s41062-018-0143-6

Hibbeler, R. C. 2012. Structutal Analysis.

Horne, M. R., And Fruchtlander, A. 1954. Correspondence. a Moment Distribution Method for the Analysis and Design of Structures By the Plastic Theory. Proceedings of the Institution of Civil Engineers, 3(4), 594–597. https://doi.org/10.1680/ipeds.1954.12588

Moy, S. S. J. 2013. Plastic Method for Steel and Concrete Strucutres. In Journal of Chemical Information and Modeling (Vol. 53, Issue 9).

Singh, A. K. 2011. Plastic deformation. Encyclopedia of Earth Sciences Series, Part 3, 864. https://doi.org/10.1007/978-90-481-2642-2_688

Stark, J. W. B., & Brekelmans, J. W. P. M. 1990. Plastic design of continuous composite slabs. Journal of Constructional Steel Research, 15(1–2), 23–47. https://doi.org/10.1016/0143-974X(90)90041-E

Vančík, V., and Jirásek, M. 2016. Computer-Aided Plastic Limit Analysis of Plates. Applied Mechanics and Materials, 821, 547–554. https://doi.org/10.4028/www.scientific.net/amm.821.547